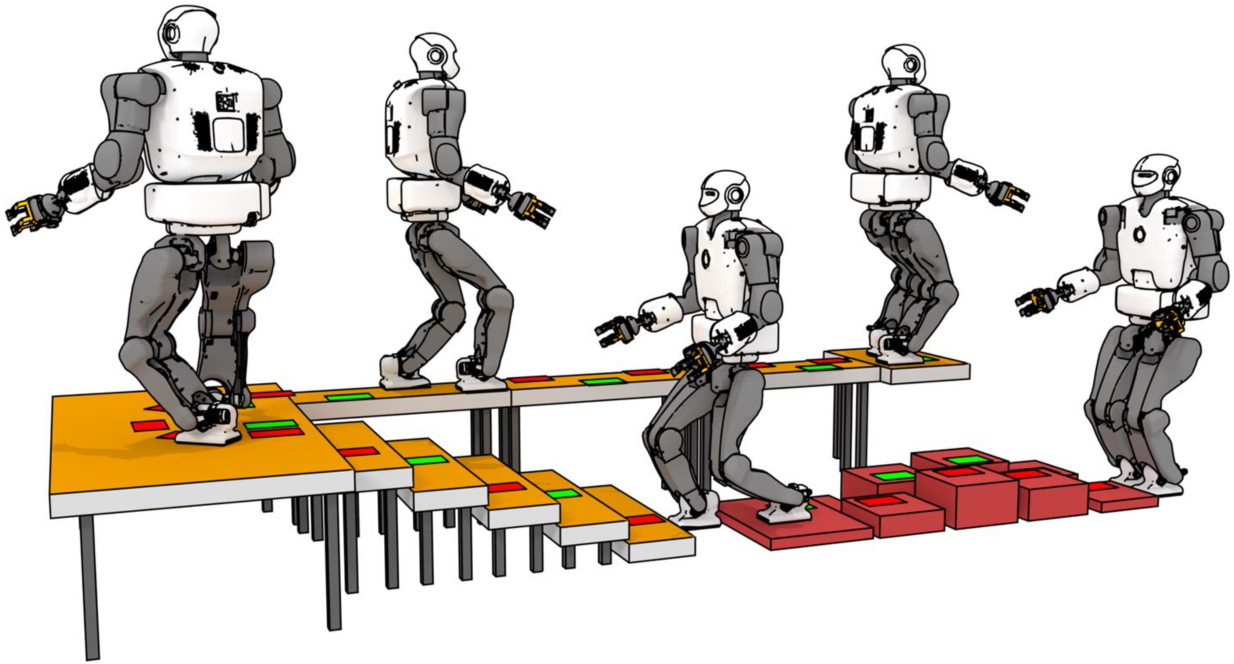

SL1M: Sparse L1-norm Minimization for contact planning on uneven terrain

Steve Tonneau - Daeun Song
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UNIVERSITÀ DEGLI STUDI DI TRENTO
Dipartimento di Ingegneria Industriale

memmo

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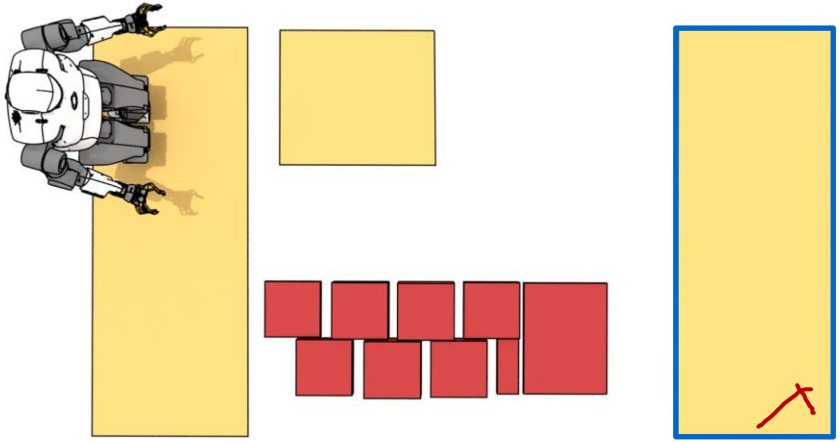
LEGGED LOCOMOTION IS DEFINED PRIMARILY BY CONTACTS

HOW TO REACH THE PLATFORM ?



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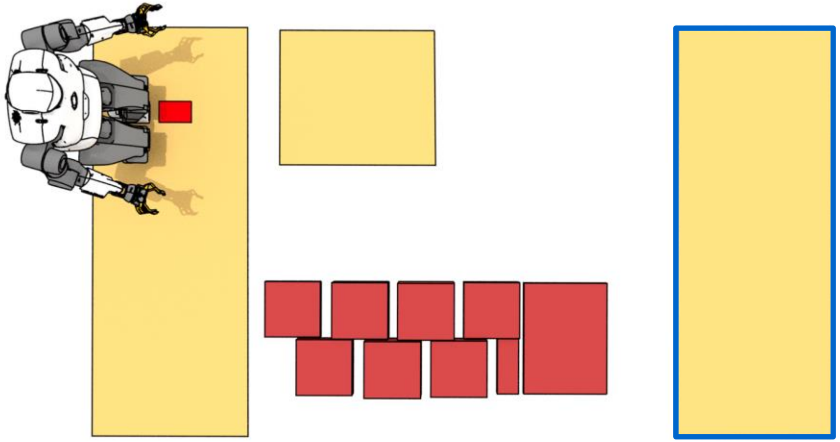
COMBINATORIAL STEP PLANNING



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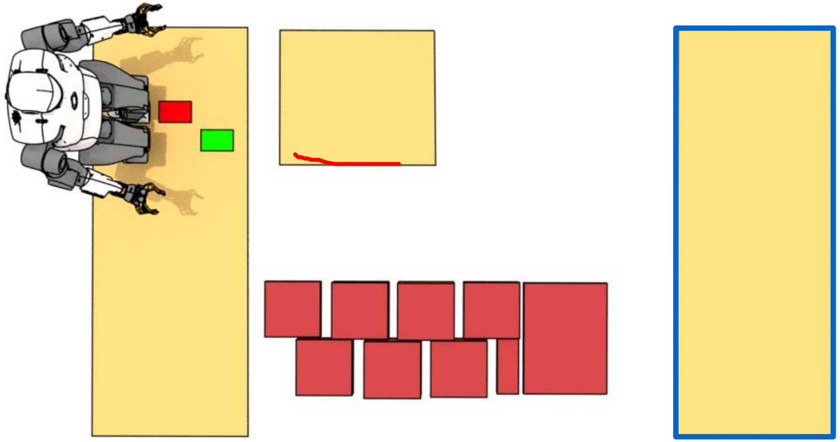
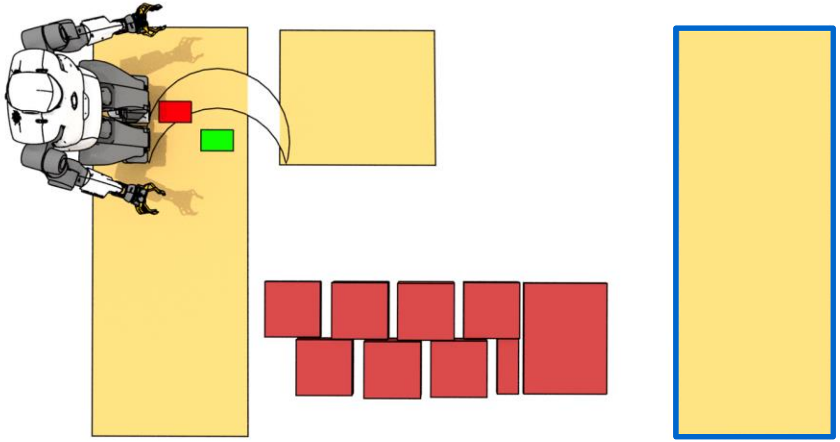
COMBINATORIAL STEP PLANNING



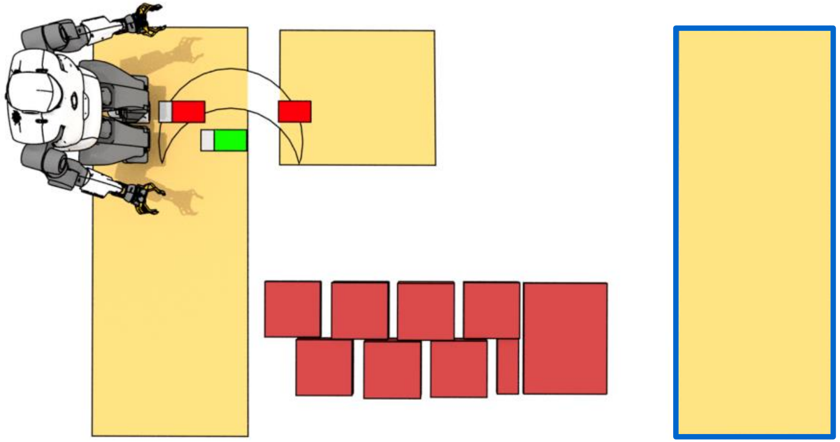
COMBINATORIAL STEP PLANNING



Feasibility $\mathcal{F}$:

<u>Geometric constraints</u>

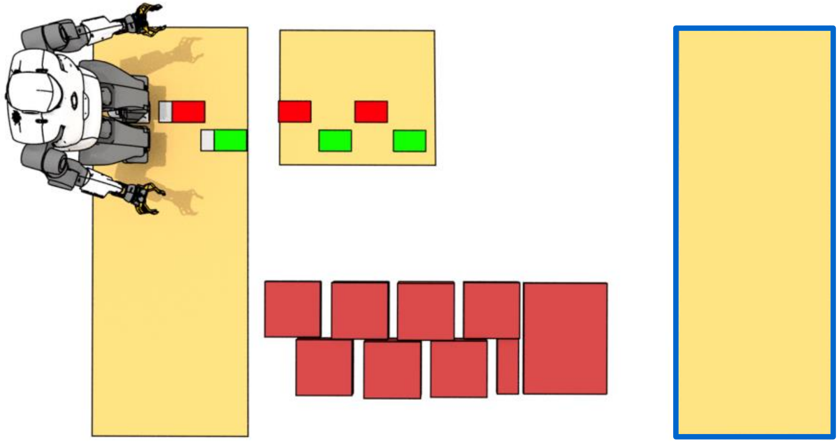
COMBINATORIAL STEP PLANNING



Feasibility $\mathcal{F}$:

<u>Geometric constraints</u>

COMBINATORIAL STEP PLANNING

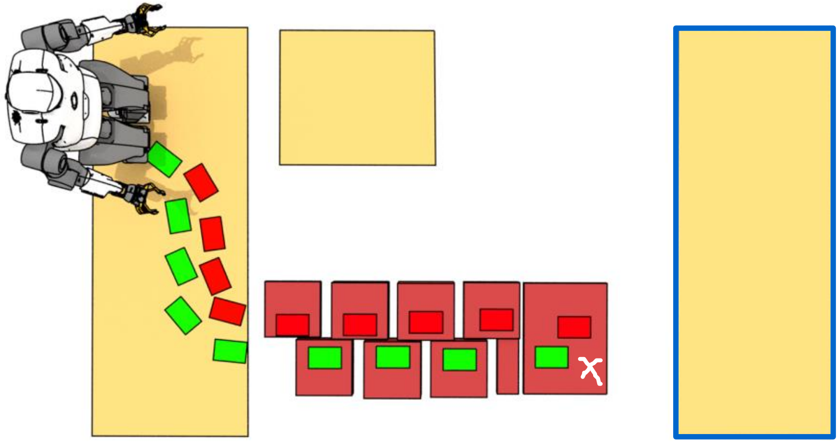


Global path search

Feasibility $\mathcal{F}$:

Geometric constraints

COMBINATORIAL STEP PLANNING

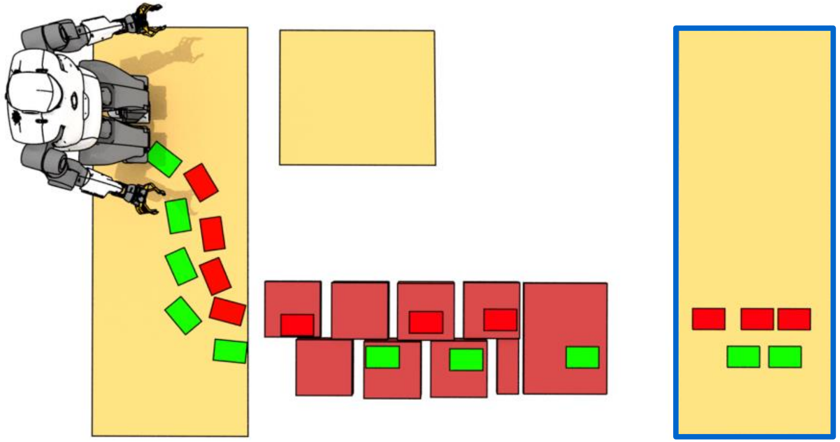


<u>Global path search</u>

Feasibility $\mathcal{F}$:

Geometric constraints

COMBINATORIAL STEP PLANNING



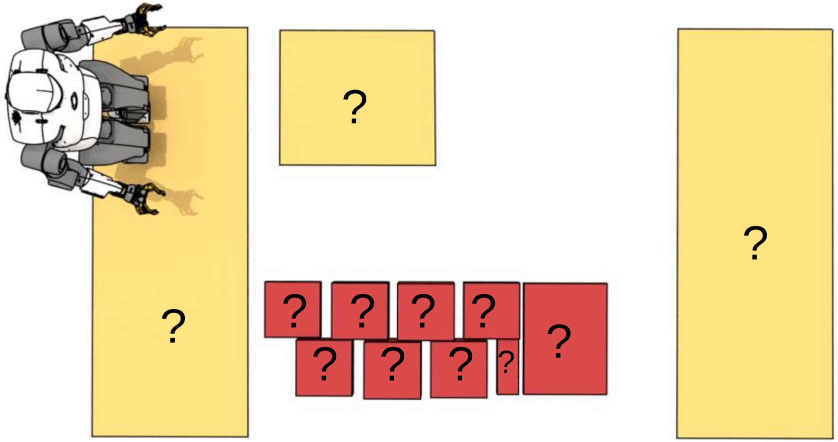
Global path search

Feasibility $\mathcal{F}$:

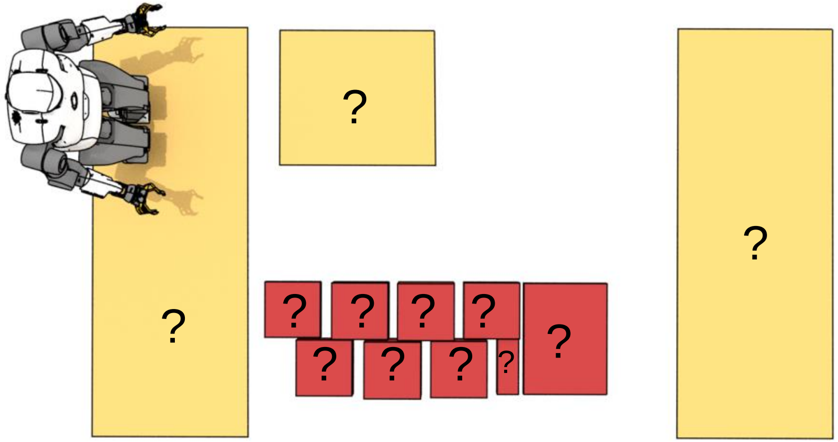
Geometric constraints

Dynamic constraints

WHICH CONTACT SURFACE ?



WHICH CONTACT SURFACE ?



Convex contact surfaces:

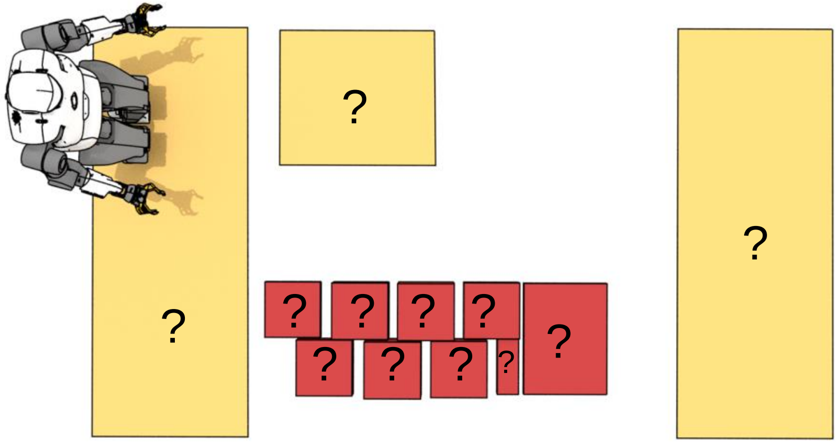
$$\mathcal{S}_j : \{\mathbf{p}, \mathbf{S}_j\mathbf{p} \leq \mathbf{s}_j\}$$

$$1 \leq j \leq n$$

Footsteps positions:

$$\mathbf{p}_i, 1 \leq i \leq m$$

WHICH CONTACT SURFACE ?



Convex contact surfaces:

$$\mathcal{S}_j : \{\mathbf{p}, \mathbf{S}_j \mathbf{p} \leq \mathbf{s}_j\}$$

$$1 \leq j \leq \underline{n}$$

Footsteps positions:

$$\mathbf{p}_i, 1 \leq i \leq \underline{m}$$

Combinatorial:

$$\underline{n^m}$$

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CONTACT PLANNING AS A FEASIBILITY PROBLEM

$$
\begin{aligned}
\text{find} \quad & \mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m] \\
\text{s.t.} \quad & \mathbf{X} \in \mathcal{F} && \text{// Feasibility} \\
& \mathbf{X} \in \mathcal{I} \cap \mathcal{G} && \text{// Initial and goal conditions} \\
& \forall i : \\
& \qquad \mathbf{p}_i \in \mathcal{S}_1 \vee \dots \vee \mathbf{p}_i \in \mathcal{S}_n
\end{aligned}
$$

CONTACT PLANNING AS A FEASIBILITY PROBLEM

$$
\begin{aligned}
\text{find} \quad & \mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m] \\
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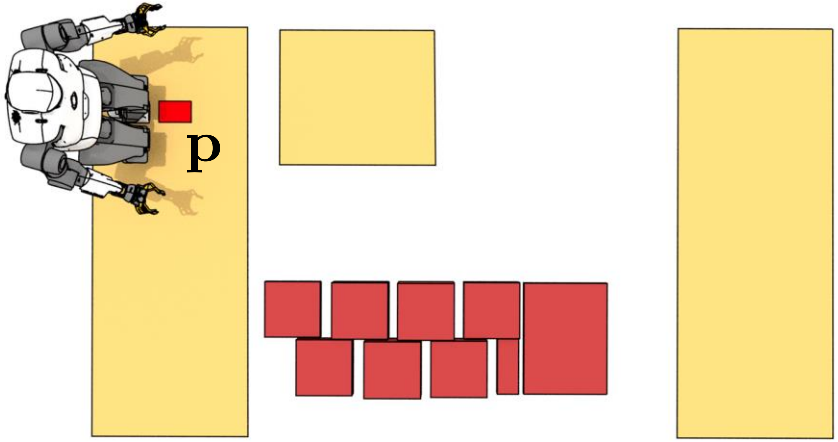
& \forall i : \\
& \qquad \mathbf{p}_i \in \mathcal{S}_1 \vee \dots \vee \mathbf{p}_i \in \mathcal{S}_n
\end{aligned}
$$

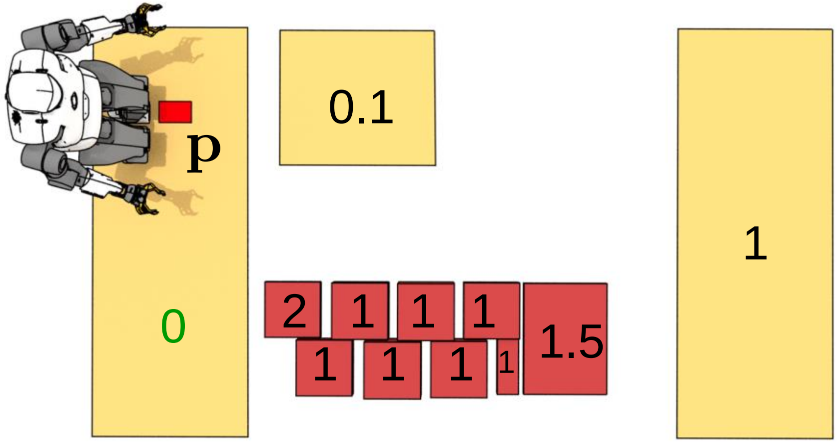
How to tackle the combinatorics ?

AN EQUIVALENT CARDINALITY MINIMIZATION PROBLEM



$$\mathbf{S}_j \mathbf{p}_i \leq \mathbf{s}_j \Leftrightarrow \mathbf{p}_i \in \mathcal{S}_j$$

AN EQUIVALENT CARDINALITY MINIMIZATION PROBLEM



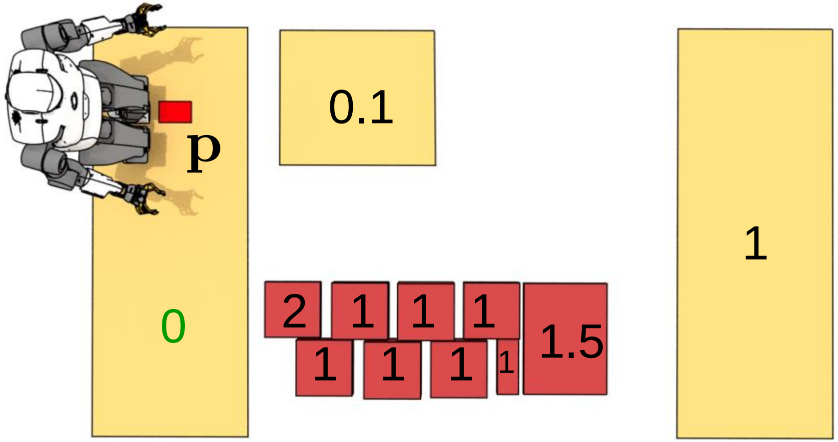
Slack variables $c_{i,j} \in \mathbb{R}^+$

$$\left.\begin{aligned} \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} &\leq \mathbf{s}_j \\ c_{i,j} &= 0 \end{aligned}\right\} \Rightarrow \mathbf{p_i} \in \mathcal{S}_j$$

$$\mathbf{c}_i = [c_{i,1}, \ldots, c_{i,n}]$$

AN EQUIVALENT CARDINALITY MINIMIZATION PROBLEM

Slack variables $c_{i,j} \in \mathbb{R}^+$



$$\left.\begin{array}{l} \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \\ c_{i,j} = 0 \end{array}\right\} \Rightarrow \mathbf{p_i} \in \mathcal{S}_j$$

$$\mathbf{c}_i = [c_{i,1}, \ldots, c_{i,n}]$$

$$\boxed{\#NonZeros(\mathbf{c}_i) = n - 1 \Rightarrow \mathbf{p}_i \quad \text{on a surface}}$$

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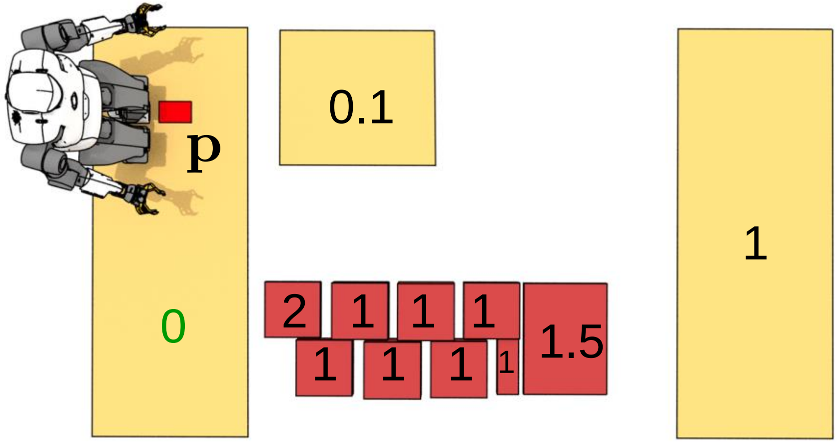
AN EQUIVALENT CARDINALITY MINIMIZATION PROBLEM



$$\begin{cases} \text{find} & \mathbf{p}_i \\ \text{s.t.} & \mathbf{p}_i \in \mathcal{S}_1 \vee \dots \vee \mathbf{p}_i \in \mathcal{S}_n \end{cases}$$

$$\Longleftrightarrow$$

$$\begin{cases} \text{find} & \mathbf{p}_i, \mathbf{c}_i \\ \text{min} & card(\mathbf{c}_i) \\ \text{s.t.} & \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \quad \forall j \end{cases}$$

AN EQUIVALENT CARDINALITY MINIMIZATION PROBLEM

$$\begin{aligned}
\text{find} \quad & \mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m] \\
\text{s.t.} \quad & \mathbf{X} \in \mathcal{F} \\
& \mathbf{X} \in \mathcal{I} \cap \mathcal{G} \\
& \forall i : \\
& \quad \boxed{\mathbf{p}_i \in \mathcal{S}_1 \vee \dots \vee \mathbf{p}_i \in \mathcal{S}_n}
\end{aligned}
\quad \Leftrightarrow \quad
\begin{aligned}
\text{find} \quad & \mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m] \\
& \mathbf{C} = [\mathbf{c}_1 \dots \mathbf{c}_n] \\
\min \quad & \sum_{i=1}^{m} \underline{card(\mathbf{c}_i)} \\
\text{s.t.} \quad & \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \forall i, \forall j \\
& \mathbf{X} \in \mathcal{F} \\
& \mathbf{X} \in \mathcal{I} \cap \mathcal{G}
\end{aligned}$$

CONVEX RELAXATION WITH A SPARSITY INDUCING NORM

[Bach et al. 11]

$$
\begin{aligned}
\text{find} \quad & \mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m] \\
& \mathbf{C} = [\mathbf{c}_1 \dots \mathbf{c}_n] \\
\min \quad & \sum_{i=1}^{m} card(\mathbf{c}_i) \\
\text{s.t.} \quad & \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \forall i, \forall j \\
& \mathbf{X} \in \mathcal{F} \\
& \mathbf{X} \in \mathcal{I} \cap \mathcal{G}
\end{aligned}
\quad \simeq \quad
\begin{aligned}
\text{find} \quad & \mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m] \\
& \mathbf{C} = [\mathbf{c}_1 \dots \mathbf{c}_n] \\
\min \quad & \sum_{i=1}^{m} \boxed{||\mathbf{c}_i||_1} \\
\text{s.t.} \quad & \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \forall i, \forall j \\
& \mathbf{X} \in \mathcal{F} \\
& \mathbf{X} \in \mathcal{I} \cap \mathcal{G}
\end{aligned}
$$

RELAXATION GIVES MUCH FASTER RESULTS… WHEN IT CONVERGES

L1 up to 100x faster than Branch and Bound…

5 steps $\leq$ 0.7 ms

What to do when relaxed problem does not converge to a sparse solution ?

FEASIBILITY VS OPTIMALITY

SL1M essentially solves a feasibility problem

Optimality of the motion not guaranteed

Does it really matter ?

THAT'S ALL FOR NOW !

https://stevetonneau.fr for paper, video, source code

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